OCR
Oxford Cambridge and RSA

Wednesday 29 June 2016 – Morning

A2 GCE MATHEMATICS

4734/01 Probability & Statistics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4734/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 On a motorway, lorries pass an observation point independently and at random times. The mean number of lorries travelling north is 6 per minute and the mean number travelling south is 8 per minute. Find the probability that at least 16 lorries pass the observation point in a given 1-minute period. **[4]**

2 A random sample of 200 American voters were asked about which political party they supported and their attitude to a proposed new form of taxation. The voters' responses are summarised in the table.

		Attitude		
		In favour	Neutral	Against
Party	Democrat	58	16	16
	Independent	25	4	11
	Republican	17	20	33

Carry out a χ^2 test, at the 1% level of significance, to investigate whether there is an association between party supported and attitude to the proposed form of taxation. **[7]**

3 **(i)** A company packages butter. Of 50 randomly selected packs, 8 were found to have damaged wrappers. Find an approximate 95% confidence interval for the proportion of packs with damaged wrappers. **[4]**

(ii) The mass of a pack has a normal distribution with standard deviation 8.5 g. In a random sample of 10 packs the masses, in g, are as follows.

220 225 218 223 224 220 229 228 226 228

Find a 99% confidence interval for the mean mass of a pack. **[4]**

4 A group of students were tested in geography before and after a fieldwork course. The marks of 10 randomly selected students are shown in the table.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Mark before fieldwork	19	84	84	99	59	19	29	49	54	69
Mark after fieldwork	23	98	83	88	68	33	28	53	58	88

(i) Use a suitable *t*-test, at the 5% level of significance, to test whether the students' performance has improved. **[8]**

(ii) State the necessary assumption in applying the test. **[1]**

5 The independent random variables X and Y have distributions N(30, σ^2) and N(20, σ^2) respectively. The random variable $aX + bY$, where a and b are constants, has the distribution N(410, $130\sigma^2$).

(i) Given that a and b are integers, find the value of a and the value of b. **[5]**

(ii) Given that $P(X > Y) = 0.966$, find σ^2. **[6]**

6 The masses at birth, in kg, of random samples of babies were recorded for each of the years 1970 and 2010. The table shows the sample mean and an unbiased estimate of the population variance for each year.

Year	No. of babies	Sample mean	Unbiased estimate of population variance
1970	285	3.303	0.2043
2010	260	3.352	0.2323

(i) A researcher tests the null hypothesis that babies born in 2010 are 0.04 kg heavier, on average, than babies born in 1970, against the alternative hypothesis that they are more than 0.04 kg heavier on average. Show that, at the 5% level of significance, the null hypothesis is not rejected. **[7]**

(ii) Another researcher chooses samples of equal size, n, for the two years. Using the same hypothesis as in part **(i)**, she finds that the null hypothesis is rejected at the 5% level of significance. Assuming that the sample means and unbiased estimates of population variance for the two years are as given in the table, find the smallest possible value of n. **[4]**

7 A continuous random variable X has probability density function

$$f(x) = \begin{cases} ax^3 & 0 \leqslant x \leqslant 1, \\ ax^2 & 1 < x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that $a = \frac{12}{31}$. **[2]**

(ii) Find $E(X)$. **[2]**

It is thought that the time taken by a student to complete a task can be well modelled by X. The times taken by 992 randomly chosen students are summarised in the table, together with some of the expected frequencies.

Time	$0 \leqslant x < 0.5$	$0.5 \leqslant x < 1$	$1 \leqslant x < 1.5$	$1.5 \leqslant x \leqslant 2$
Observed frequency	8	92	279	613
Expected frequency	6	90		

(iii) Find the other expected frequencies and test, at the 5% level of significance, whether the data can be well modelled by X. **[8]**

8 The radius, R, of a sphere is a random variable with a continuous uniform distribution between 0 and 10.

(i) Find the cumulative distribution function and probability density function of A, the surface area of the sphere. **[8]**

(ii) Find $P(A \leqslant 200\pi)$. **[2]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1			Poisson identified or implied. Mean = 14 1 – 0.6694 0.331	B1 B1 M1 A1 **[4]**	If N used, B0.	
2			H_0:there is no assoc. between party and opinion, H_1:there is assoc. between p/o. Expected frequencies 45, 18, 27, 20, 8, 12, 35, 14, 21 $\frac{(58-45)^2}{45} + \ldots + \frac{(33-21)^2}{21}$ 30.48 TS > 13.28, reject H_0 There is evidence that there is an association between party and opinion.	B1 M1 A1 M1 A1 M1 A1 **[7]**	For both.Allow indpt. etc. At least one correct term; at least 7 terms. Allow awrt 30.5 CWO	If classes combined, all 6.
3	(i)		Var = $\frac{8\times42}{50^3}$ 1.96 $0.16 \pm 1.96\sqrt{\frac{8\times42}{50^3}}$ [0.058,0.262]	B1 B1 M1 A1 **[4]**	Var must be of correct structure. Allow 2sf. Must be interval.	
3	(ii)		$\bar{x} = 224.1$ 2.576 "224.1" $\pm 2.576 \times \frac{8.5}{\sqrt{10}}$	B1 B1 M1	Any z. NOT t. Must be correct sd.	Must see all values substituted.

Question			Answer	Marks	Guidance	
			[217.2,231.0]	A1 **[4]**	Allow [217,231] Must be interval.	Do not penalise if penalised in i.
4	(i)		$H_0: \mu_1 = \mu_2$ $H_1: \mu_2 > \mu_1$	B1	allow $\mu_D=0$, $\mu_D>0$; $\delta=0$, $\delta>0$. If words, "population" must be used.	2-sample test.
			Diffs 4, 14, -1, -11, 9, 14, -1, 4, 4, 19	M1	allow all reversed. Must have differences of both signs. Can be implied by attempt at s.	B1 Hyps.
			$\bar{d} = 5.5$	A1	allow -5.5	B1 5.5 or 62–56.5
			$\hat{\sigma} = 8.83(49....)$	A1	$\hat{\sigma}^2 = \frac{1405}{18} = 78.05$	M1 5.5/σ,σ must include 1/10 or 1/10 + 1/10
			TS = $\frac{5.5}{\left(\frac{8.835}{\sqrt{10}}\right)}$	M1	allow if s used. Must have $\sqrt{10}$. SC $\bar{d} = 5.5$ no diffs seen B1.	
			1.97	A1		
			"1.97" > 1.833 reject H_0	M1	Allow consistent –ve signs.	M1 compare with 1.734, make correct 1st conclusion.
			There is evidence that the field-work has improved performance.	A1 **[8]**	Contextualised, not over-assertive. CWO.	Max 4 out of 8.
	(ii)		Differences are normally distributed.	B1 **[1]**	allow if seen. Ignore all else.	
5	(i)		$30a + 20b = 410$ oe $a^2 + b^2 = 130$ oe $13a^2 – 246a + 1161 = 0$ $a = 9$, $b = 7$	B1 B1 M1A1 A1 **[5]**	Obtain quadratic in a or b.	$13b^2 – 164b + 511=0$

Question			Answer	Marks	Guidance	
	(ii)		$X - Y \sim \mathrm{N}(10, 2\sigma^2)$	M1A1	M1 for N(10,anything)	allow -10,30-20,20-30 for M1
			$\frac{0-10}{\sqrt{2\sigma^2}}$ =-1.825 (or 6)	M1A1	must have $2\sigma^2$.Must have matching signs for M1..	
			(-)1.825/6 seen	B1		

Question			Answer	Marks	Guidance	
			15.0 CWO	A1 **[6]**	$\frac{80000}{5329}$	
6	(i)		H_0: $\mu_y - \mu_x = 0.04$ H_1: $\mu_y - \mu_x > 0.04$ $\hat{\sigma}^2 = \sqrt{\frac{0.2043}{285} + \frac{0.2323}{260}}$ $\frac{3.352-3.303-0.04}{\hat{\sigma}}$ 0.224 CV = 1.645 TS < CV ,(so do not reject H_0)	B1 M1A1 M1 A1 B1 M1 **[7]**	Allow ≤ in NH, allow μ_D etc. M1 for attempt to combine vars using denominators, inc pooled. ±0.009 soi. p=0.411 Must be <, AG. ft TS or prob. 0.411>0.05oe do not reject H_0.	Allow 284, 259 NB 0.04 missing →1.221
	(ii)		$\frac{0.009}{\sqrt{\frac{0.2043}{n} + \frac{0.2323}{n}}} > 1.645$ Solve 14 586	M1A1 M1 A1 **[4]**	allow =, allow 3.352-3.303-0.04. Correct var only. Allow 14 600 or 14590. Must be integer.	Incorrect z ,M1A0. NB 0.04 missing→492/3.M0A0M1A0. NOT 14585.(not ISW)
7	(i)		$\int_0^1 ax^3 dx + \int_1^2 ax^2 dx = 1$ $a = \frac{12}{31}$ AG	M1 A1 **[2]**	Need to see $a/4$ & $7a/3$ oe	

Question			Answer	Marks	Guidance	
7	(ii)		$\int_0^1 ax^4 dx + \int_1^2 ax^3 dx$ $\frac{237}{155}$ or 1.53	M1 A1 **[2]**	allow x.ax³, x.ax²	
7	(iii)		H_0: data can be modelled by X H_1: data cannot be modelled by X oe Expected values [6, 90,] 304, 592 $TS = \frac{(8-6)^2}{6} + ... + \frac{(613-592)^2}{592}$ = 3.51(2) TS < 7.815, do not reject H_0 Insufficient evidence that data cannot be modelled by X.	B1 M1A2 M1 A1 M1 A1 **[8]**	Or f(x) M1A1 for 1 correct ft TS not CV. Allow 7.8 CWO	eg $992 \times \frac{12}{31}\left[\frac{x^3}{3}\right]_1^{1.5}$ for M1
8	(i)		pdf of $R = \frac{1}{10}$ soi	B1	Method based on int. by subn. 1st B1 as main scheme. $\int_0^{10} \frac{1}{10} dr = 1$ M1* $\frac{dA}{dr} = 8\pi r$ *M1 $\int_0^{400\pi} \frac{1}{80\pi} \frac{dA}{\sqrt{\frac{A}{4\pi}}} = 1$ M1 pdf= $\frac{1}{40\sqrt{\pi a}}$ B1 attempt int to obtain cdf. M1	NOT 8πr alone

Question			Answer	Marks	Guidance	
			Cdf of $R = \frac{r}{10}$ allow $\frac{x}{10}$ $[F_A(a) =] P(A \le a) = P(4\pi R^2 \le a)$ $P(R \le \sqrt{[a/4\pi]}) = F_R(\sqrt{[a/4\pi]})$ $\frac{\sqrt{a}}{20\sqrt{\pi}}$ (SC B1 if obtained without M marks) $f_A(a) = \frac{1}{40\sqrt{a\pi}}$ $0 \le a \le 400\pi$	B1 M1 M1 A1 M1A1 B1 **[8]**	$\frac{\sqrt{a}}{20\sqrt{\pi}}$ A1 final B1 for range as main scheme. allow M0M1 for eg πr^2 rearranged and substituted correctly. M1 for diffn.	

Question			Answer	Marks	Guidance	
8	(ii)		$\frac{\sqrt{200\pi}}{20\sqrt{\pi}}$	M1	Sub 200π in their cdf.	$4\pi r^2=200\pi$ $r=\sqrt{50}$ prob$=\frac{\sqrt{50}}{10}$ M1
			0.707 or $\frac{1}{\sqrt{2}}$	A1 **[2]**		0.707 A1